

# UNIVERSITY OF TECHNOLOGY, JAMAICA

### FACULTY OF SCIENCE AND SPORT

## SCHOOL OF MATHEMATICS AND STATISTICS

**REDO EXAMINATION, SEMESTER 3** 

Module Name:

**Engineering Mathematics 3** 

Module Code:

**MAT 2034** 

Date:

July 2010

Theory/Practical:

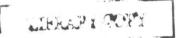
Theory

Group:

Eng 2

**Duration:** 

2 hours



#### Instructions:

- 1. Candidates are required to answer any four (4) questions.
- 2. This examination paper consists of five questions, each worth 15 marks.
- 3. All working must be clearly shown.
- 4. Begin the answer to each question on a new page.
- 5.  $\mathbb{R}$  is the set of real numbers.
- 6. The use of silent electronic calculators is allowed.

#### QUESTION # 1

- (a) If  $Z = ye^{2x} xe^{2y}$ , show that:
  - (i)  $Z_x + Z_y = 2Z + \frac{1}{2}Z_{xy}$
  - (ii)  $Z_{xx} + Z_{yy} = 4Z$
- (b) Given the parametric equations:  $y = \theta \ln \theta$ ,  $x = e^{3\theta^2 3}$ , show that the gradient of the <u>normal</u> at  $\theta = 1$  equals -6. [5 marks]

#### QUESTION # 2

- (a) If  $W = \frac{Ax^2}{h^3 \sqrt{y}}$ , where W = f(h, x, y), find the approximate percentage change in W, given that 'h' decreases by 3% and both 'x' and 'y' increase by 2% and 4% respectively. [Use partial differentiation.] [4 marks]
- (b) For the parametric curve:  $y = e^{\cos t}$ ,  $x = \cos t$ , show that  $\frac{d^2y}{dx^2} = y$ . [4 marks]
- (c) Evaluate the following integrals:

(i) 
$$\int_{0}^{2} \left(\theta^{2} - \frac{3}{\theta}\right)^{2} d\theta$$
  
(ii)  $\int_{0}^{1} \frac{3}{\sqrt{(5x+4)}} dx$  [4+3 marks]

#### **QUESTION # 3**

(a) Find the following integrals, simplifying your answers:

(i) 
$$\int \left[ \frac{5}{\sqrt[3]{(4-100t)^4}} - \frac{21}{2} \cos\left(\frac{4t}{7}\right) + \frac{11}{e^{\frac{9-10t}{5}}} \right] dt$$

(ii) 
$$\int \left[ \frac{9x^3 - 10x^4}{11x^4} + \frac{x}{x^2 + 7} - \frac{1}{3x^7} \right] dx$$
 [5+4 marks]

(b) Show that the gradient of the <u>tangent</u> at the point  $\left(1, \frac{\pi}{2}\right)$  on the curve  $\ln(2x-1) - 10y + xe^{\cos y} + 5\pi - 1 = 0 \text{ equals } \frac{3}{11}.$  [6 marks]

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#### QUESTION # 4

- (a) (i) Find the two stationary points of the function:  $Z = 6x^2 3xy + y^3 2$ .
  - (ii) For each critical point, determine by the second partial derivative test, whether it corresponds to a relative maximum, relative minimum or saddle point.

[6+4 marks]

(b) If 
$$y = \left[\frac{k x^{mx+n} \cdot \cot^2 x}{e^{\sin 6x} \cdot \sqrt[k]{(7x^5 + x)}}\right]$$
, where  $k, m, n \in \mathbb{R}$ , find  $\frac{dy}{dx}$ .

[ Hint: Use logarithmic differentiation ]

[5 marks]

#### QUESTION # 5

- (a) Differentiate the following with respect to x, simplifying your answers as far as possible:
  - (i)  $100^{x-10} (x-10)^{100}$
  - (ii)  $\sin^{-1}\left(\sqrt{x^7}\right) \ln\left(e^{\tan x} x\right)$
  - (iii)  $\cos^9\left(\frac{5}{x-1}\right)$

[ 4+4+3 marks]

(b) The intensity of an earthquake is measured on the Richter scale. The reading R is given by  $R = \log\left(\frac{I}{I_0}\right)$ , where I is the intensity and  $I_0$  is a standard minimum intensity. If  $I_0 = 12$ , find  $\frac{dR}{dI}$ , the rate of change of the Richter scale reading with respect to the intensity. [4 marks]

\*\*\*\*\*\*\* END OF PAPER \*\*\*\*\*\*\*

#### SOME USEFUL RULES

If 
$$Z = f(t, x, y)$$
 then,  $\Delta Z \approx \frac{\partial Z}{\partial t} \Delta t + \frac{\partial Z}{\partial x} \Delta x + \frac{\partial Z}{\partial y} \Delta y$ 

$$\ln \left[ \frac{e^k a^m}{b c^p} \right] = k + m \ln a - \ln b - p \ln c$$

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