UNIVERSITY OF TECHNOLOGY, JAMAICA

FACULTY OF SCIENCE AND SPORT

SCHOOL OF MATHEMATICS AND STATISTICS

REDO EXAMINATION, SEMESTER 3

Module Name:	**Engineering Mathematics 3**
Module Code:	**MAT 2034**
Date:	**July 2010**
Theory/Practical:	**Theory**
Group:	**Eng 2**
Duration:	**2 hours**

Instructions:

1. Candidates are required to answer <u>any four (4) questions</u>.
2. This examination paper consists of five questions, each worth 15 marks.
3. All working must be clearly shown.
4. Begin the answer to each question on a new page.
5. $\mathbb{R}$ is the set of real numbers.
6. The use of silent electronic calculators is allowed.

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO

QUESTION # 1

(a) If $Z = ye^{2x} - xe^{2y}$, show that:

(i) $Z_x + Z_y = 2Z + \frac{1}{2} Z_{xy}$

(ii) $Z_{xx} + Z_{yy} = 4Z$ **[6 + 4 marks]**

(b) Given the parametric equations: $y = \theta \ln \theta$, $x = e^{3\theta^2 - 3}$,

show that the gradient of the normal at $\theta = 1$ equals -6. **[5 marks]**

QUESTION # 2

(a) If $W = \dfrac{Ax^2}{h^3 \sqrt{y}}$, where $W = f(h, x, y)$, find the approximate percentage change in W, given that 'h' decreases by 3% and both 'x' and 'y' increase by 2% and 4% respectively. **[Use partial differentiation.]** **[4 marks]**

(b) For the parametric curve: $y = e^{\cos t}$, $x = \cos t$, show that $\dfrac{d^2y}{dx^2} = y$. **[4 marks]**

(c) Evaluate the following integrals:

(i) $\displaystyle\int_1^2 \left(\theta^2 - \frac{3}{\theta}\right)^2 d\theta$

(ii) $\displaystyle\int_0^1 \frac{3}{\sqrt{(5x+4)}}\, dx$ **[4+3 marks]**

QUESTION # 3

(a) Find the following integrals, simplifying your answers:

(i) $\displaystyle\int \left[\frac{5}{\sqrt[3]{(4-100t)^4}} - \frac{21}{2}\cos\left(\frac{4t}{7}\right) + \frac{11}{e^{\frac{9-10t}{5}}} \right] dt$

(ii) $\displaystyle\int \left[\frac{9x^3 - 10x^4}{11x^4} + \frac{x}{x^2+7} - \frac{1}{3x^7} \right] dx$ **[5+4 marks]**

(b) Show that the gradient of the tangent at the point $\left(1, \frac{\pi}{2}\right)$ on the curve

$\ln(2x-1) - 10y + xe^{\cos y} + 5\pi - 1 = 0$ equals $\frac{3}{11}$. **[6 marks]**

GO ONTO NEXT PAGE

QUESTION # 4

(a) (i) Find the two stationary points of the function: $Z = 6x^2 - 3xy + y^3 - 2$.

(ii) For each critical point, determine by the second partial derivative test, whether it corresponds to a relative maximum, relative minimum or saddle point.

[6+4 marks]

(b) If $y = \left[\dfrac{k\, x^{mx+n} . \cot^2 x}{e^{\sin 6x} . \sqrt[k]{\left(7x^5 + x\right)}} \right]$, where $k, m, n \in \mathbb{R}$, find $\dfrac{dy}{dx}$.

[Hint: Use logarithmic differentiation] **[5 marks]**

QUESTION # 5

(a) Differentiate the following with respect to x, simplifying your answers as far as possible:

(i) $100^{x-10} - (x-10)^{100}$

(ii) $\sin^{-1}\left(\sqrt{x^7}\right) - \ln\left(e^{\tan x} - x\right)$

(iii) $\cos^9\left(\dfrac{5}{x-1}\right)$ **[4+4+3 marks]**

(b) The intensity of an earthquake is measured on the Richter scale. The reading R is given by $R = \log\left(\dfrac{I}{I_0}\right)$, where I is the intensity and I_0 is a standard minimum intensity. If $I_0 = 12$, find $\dfrac{dR}{dI}$, the rate of change of the Richter scale reading with respect to the intensity. **[4 marks]**

********** **END OF PAPER** **********

SOME USEFUL RULES

If $Z = f(t, x, y)$ *then,* $\Delta Z \approx \dfrac{\partial Z}{\partial t}.\Delta t + \dfrac{\partial Z}{\partial x}.\Delta x + \dfrac{\partial Z}{\partial y}.\Delta y$

$$\ln\left[\frac{e^k a^m}{b c^p}\right] = k + m \ln a - \ln b - p \ln c$$