



UNIVERSITY OF TECHNOLOGY, JAMAICA

FACULTY OF SCIENCE AND SPORT

SCHOOL OF MATHEMATICS AND STATISTICS

REDO EXAMINATION, SEMESTER 3

Module Name: Engineering Mathematics 3

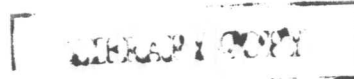
Module Code: MAT 2034

Date: July 2010

Theory/Practical: Theory

Group: Eng 2

Duration: 2 hours



Instructions:

1. Candidates are required to answer any four (4) questions.
2. This examination paper consists of five questions, each worth 15 marks.
3. All working must be clearly shown.
4. Begin the answer to each question on a new page.
5. \mathbb{R} is the set of real numbers.
6. The use of silent electronic calculators is allowed.

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO

QUESTION # 1

(a) If $Z = ye^{2x} - xe^{2y}$, show that:

(i) $Z_x + Z_y = 2Z + \frac{1}{2}Z_{xy}$

(ii) $Z_{xx} + Z_{yy} = 4Z$

[6 + 4 marks]

(b) Given the parametric equations: $y = \theta \ln \theta$, $x = e^{3\theta^2-3}$,

show that the gradient of the normal at $\theta = 1$ equals -6 .

[5 marks]

QUESTION # 2

(a) If $W = \frac{Ax^2}{h^3\sqrt{y}}$, where $W = f(h, x, y)$, find the approximate percentage change in W , given that 'h' decreases by 3% and both 'x' and 'y' increase by 2% and 4% respectively.

[Use partial differentiation.]

[4 marks]

(b) For the parametric curve: $y = e^{\cos t}$, $x = \cos t$, show that $\frac{d^2y}{dx^2} = y$.

[4 marks]

(c) Evaluate the following integrals:

(i) $\int_0^2 \left(\theta^2 - \frac{3}{\theta} \right)^2 d\theta$

(ii) $\int_0^1 \frac{3}{\sqrt{(5x+4)}} dx$

[4+3 marks]

QUESTION # 3

(a) Find the following integrals, simplifying your answers:

(i) $\int \left[\frac{5}{\sqrt[3]{(4-100t)^4}} - \frac{21}{2} \cos\left(\frac{4t}{7}\right) + \frac{11}{e^{\frac{9-10t}{5}}} \right] dt$

(ii) $\int \left[\frac{9x^3 - 10x^4}{11x^4} + \frac{x}{x^2 + 7} - \frac{1}{3x^7} \right] dx$

[5+4 marks]

(b) Show that the gradient of the tangent at the point $\left(1, \frac{\pi}{2}\right)$ on the curve

$\ln(2x-1) - 10y + xe^{\cos y} + 5\pi - 1 = 0$ equals $\frac{3}{11}$.

[6 marks]

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QUESTION # 4

- (a) (i) Find the two stationary points of the function: $Z = 6x^2 - 3xy + y^3 - 2$.
- (ii) For each critical point, determine by the second partial derivative test, whether it corresponds to a relative maximum, relative minimum or saddle point.

[6+4 marks]

(b) If $y = \left[\frac{k x^{mx+n} \cdot \cot^2 x}{e^{\sin 6x} \cdot k \sqrt{(7x^5 + x)}} \right]$, where $k, m, n \in \mathbb{R}$, find $\frac{dy}{dx}$.

[Hint: Use logarithmic differentiation]

[5 marks]

QUESTION # 5

- (a) Differentiate the following with respect to x , simplifying your answers as far as possible:

(i) $100^{x-10} - (x-10)^{100}$

(ii) $\sin^{-1}(\sqrt{x^7}) - \ln(e^{\tan x} - x)$

(iii) $\cos^9\left(\frac{5}{x-1}\right)$

[4+4+3 marks]

- (b) The intensity of an earthquake is measured on the Richter scale. The reading R is given by $R = \log\left(\frac{I}{I_0}\right)$, where I is the intensity and I_0 is a standard minimum intensity. If $I_0 = 12$, find $\frac{dR}{dI}$, the rate of change of the Richter scale reading with respect to the intensity.

[4 marks]

***** **END OF PAPER** *****

SOME USEFUL RULES

If $Z = f(t, x, y)$ then, $\Delta Z \approx \frac{\partial Z}{\partial t} \Delta t + \frac{\partial Z}{\partial x} \Delta x + \frac{\partial Z}{\partial y} \Delta y$

$$\ln\left(\frac{e^k a^m}{b c^p}\right) = k + m \ln a - \ln b - p \ln c$$

