BASIC HEAT TRANSFER AND SOME APPLICATIONS IN POLYMER PROCESSING

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John Vlachopoulos and David Strutt
www.polydynamics.com

Heat transfer is a branch of engineering science which seeks to determine the <u>rate of energy transfer between bodies as a result of temperature differences</u>. The concept of <u>rate</u> is the basic difference between heat transfer and thermodynamics. Thermodynamics deals with systems in equilibrium and is concerned with the amount of heat required to change a system from one state to another. Thermodynamics does not answer the question of "how fast" a change is accomplished. It is the science of heat transfer that deals with this question.

BASIC CONCEPTS

The terms <u>temperature</u> and <u>heat</u> are understood intuitively. Temperature is the physical property that describes whether a body is "hot or cold". If we touch a hot metal at 120°F (49°C) heat flows from the metal to our hand. If we touch a cold block of ice heat will flow from our hand to the ice. So, the intuitive concept of temperature is really defined by the <u>heat exchange</u> from one body to another. Heat is a form of energy that flows from one body to another as a result of a temperature difference.

The two temperature scales used for measurement purposes are the <u>Fahrenheit</u> (°F) and <u>Celsius</u> (°C) scales. These scales were established by specifying the number of increments between the freezing (32°F, 0°C) and boiling point (212°F, 100°C) of water at standard atmospheric pressure. The absolute Celsius scale is called the Kelvin (K) and the absolute Fahrenheit scale is termed Rankine (R). The following conversion relations apply

$$°F = 32 + \frac{9}{5}°C$$

$$°C = \frac{5}{9}(°F - 32)$$

$$R = \frac{9}{5}K$$

$$R = °F + 459.69$$

$$K = °C + 273.16$$

In other words, 0 degrees R = **-**459.69°F and 0 degrees K = **-**273.16°C (absolute zero).

Temperature is measured by observing its effect on some easily observable property of a measuring device e.g. expansion of mercury in a glass thermometer.

Heat, or energy in general, is usually measured in Btu, cal, kcal and Joule (J). 1 Btu will raise 1 lbm of water 1°F at 68°F, 1 cal will raise 1 g of water 1°C at 20°C, 1 kcal will raise 1 kg of water 1°C at 20°C. The definition of Joule (J) comes from the definition of <u>work</u> done by a force on an object: (Newton · meter = N · m = J)

$$\text{Work} \quad W = F \cdot S$$

where F is the force and S the distance travelled. <u>Power</u> is the work done per second and J/s is called Watt (W)

$$\text{Power} = F \cdot \frac{S}{t} = F \cdot V$$

where t is time and V the velocity of travel. If we multiply Watts by time, the result is work done. The work done by electricity is usually expressed as kilowatt hours, 1000 Watts times 3600 seconds, or 3.6×10^6 Joules.

The following conversion relations apply:

1 Btu = 1055 J

1 kcal = 4182 J

1 Btu = 252 cal

$$1\,W = 1\frac{J}{s} = 3.413\,\text{Btu/hr}$$

The human metabolism requires about 2500 kcal per day (roughly 400 Btu per hour) according to frequently quoted guidelines for daily food consumption.

FUNDAMENTAL THERMODYNAMIC LAWS

The <u>First Law of Thermodynamics</u> is the principle of conservation of energy, which states that energy cannot be created nor destroyed. A more technical definition is:

> The increase of internal energy (ΔE) of a given system is equal to heat (Q) absorbed from the surroundings plus the mechanical work (W) added.

$$Q + W = \Delta E$$

If no heat enters or leaves the system the process is referred to as <u>adiabatic</u> thus

$$\Delta E = W$$

On the other hand, if no work is done by the surroundings, then

$$\Delta E = Q$$

Heat and work are different types of energy. Heat capacity C_p (also called Specific Heat) is the amount of heat required to raise the temperature of a body by one degree. Here are some typical values:

Water (at 20°C, 68°F) = 1 kcal/kg°C = 4182 J/kg°C
Air (approximately) = 0.239 kcal/kg°C = 1000 J/kg°C
Polyethylene (approximately) = 0.550 kcal/°C = 2300 J/kg°C
Steel (approximately) = 0.108 kcal/°C = 450 J/kg°C
Brick (approximately) = 0.215 kcal/°C = 900 J/kg°C

For a melting of a solid heat must be added to shake and demolish the crystal structure present. Heat of fusion (ΔH_f) is the amount of heat required to melt a crystalline solid without raising its temperature. It is equal in magnitude (but opposite in sign) to the heat of crystallization. Here are some typical values:

Ice ΔH_f = 333,000 J/kg = (80 kcal/kg)

This means that to melt 1 kg of water we need the same amount of energy as that required to raise 1 kg of water by 80°C, and yet with the actual melting, there is no increase in temperature.

HDPE ΔH_f = 250,000 J/kg
LDPE = ΔH_f = 200,000 J/kg

For amorphous polymers like PS, PMMA and PC $\Delta H_f = 0$, since they do not have any crystal structure.

Heat of reaction is the amount of heat involved in a chemical reaction (added or removed). All polymerization reactions (production of polymers) are exothermic, i.e. they involve liberation of heat.

The Second Law of Thermodynamics relates to the direction in which energy transfer or conversion may take place. The result is the increase of Entropy, which is a loss in the availability of energy for external purposes. Heat will flow from a high temperature to a low temperature. It is impossible to construct a machine or device which will operate continuously by receiving heat from just a single reservoir and producing work.

A heat engine is a device that produces net positive work as a result of heat transfer from a high-temperature body to a low-temperature body. The thermal efficiency of such engines is defined as the ratio of output (energy sought) to the input (energy that costs). The input or the energy that costs is the heat from the high temperature source (indirectly, the cost for the fuel)

We define thermal efficiency as

$$\text{Thermal efficiency } \eta = \frac{W(\text{energy sought})}{Q \ (\text{energy that costs})}$$

The efficiency of heat engines is higher if the heat source has higher temperature. However, all such actual devices have low efficiencies (e.g. combustion engines for cars, no more than 35%). In other words, only about 1/3 of the energy in gasoline goes to useful work (motion of the car). The rest is wasted due to thermodynamic implications. This is the reason why there is so much research on fuel cells nowadays, in which fuels react with oxygen to produce electricity, at higher efficiencies (50–60%).

For more information on the thermodynamic laws and their implications the reader is referred to more specialized textbooks [1-3].

Example 1

Thirty people gather for a cocktail party in a basement room that can be assumed completely sealed off and insulated. The room dimensions are 24 ft × 28 ft with 8 ft ceiling. Calculate the temperature rise in 30 minutes.

Solution

We apply the first law of thermodynamics

$$Q + W = \Delta E$$

Since there is no work being added to the air in the room

$$\Delta E = Q$$

We will assume that each person gives off approximately 2500 kcal/day (equal to the average metabolic energy consumption for light activity, like talking and walking around the room)

$$\Delta E = Q = 30 \times 2500 = 75,000 \text{ kcal/day} = 3125 \text{ kcal/hour}$$
$$= 52 \text{ kcal/min}$$

This amount of energy goes to the air inside the room which is roughly

$$\text{Air volume} = 24 \text{ ft} \times 28 \text{ ft} \times 8 \text{ ft} = 5376 \text{ ft}^3 = 152.25 \text{ m}^3$$

We neglect the volume occupied by the people.

The density of air is about 1.24 kg/m^3. So the total mass of air in the room is

$$m = 1.24 \times 152.25 = 188.79 \text{ kg.}$$

The internal energy change will be

$$\Delta E = m\ C_p\ \Delta T$$

where C_p is the heat capacity of air (0.239 kcal/°C).

$$\Delta T = \frac{\Delta E}{m\ C_p} = \frac{52\ \text{kcal/min}}{188.79\ \text{kg} \times 0.239\ \text{kcal/kg}\,°C} = 1.15°C\ /\ \text{min} = 2.07°F\ /\ \text{min}$$

So, in 30 minutes the temperature would rise by 30 × 1.15°C/min = 34.5°C (62.1°F)! This means that even a cold room would become quickly very hot, if the assumption of complete insulation is valid. In reality, there would be considerable heat losses to the surroundings that will slow down the temperature rise.

Example 2

In an injection molding machine 20 kg of LDPE are molded per hour. The melt temperature entering the mold is 180°C and the mold temperature is maintained at 40°C by a cooling water system. Determine the amount of water required to cool the plastic and keep the mold at 40°C, if the difference in input-output temperatures of the water is not to exceed 5°C.

Solution

This is a straightforward application of the first law of thermodynamics, that is the principle of conservation of energy. The heat for coming off the solidifying plastic in the mold must be taken away by the water.

$$Q_{\text{plastic}} = Q_{\text{water}}$$

The heat removed from the plastic is equal to the heat given off as the plastic temperature drops from 180°C to 40°C plus the heat of solidification which is equal to the heat of fusion but opposite in sign. As the plastic solidifies and the molecules stop moving randomly, heat is liberated.

$$Q_{\text{plastic}} = \dot{m}\ c_p\ \Delta T + \dot{m}\ \Delta H_f$$

$\dot{m}$ is amount of material molded per hour, C_p its heat capacity and ΔH_f the heat of solidification (200,000 J/kg for LDPE which solidifies around 106°C).

$$Q_{\text{plastic}} = 20\ \frac{\text{kg}}{\text{hr}}\ \frac{\text{hr}}{3600\,\text{s}} \times 2300\ \frac{\text{J}}{\text{kg}\,°C} \times (180° - 40°\,C)$$
$$+ 20\ \frac{\text{kg}}{\text{hr}}\ \frac{\text{hr}}{3600\,\text{s}} \times 200,000\ \text{J/kg}$$
$$= 1789 + 1111 = 2900\ \frac{\text{J}}{\text{s}}$$

The heat taken up by the water undergoing a 5°C temperature change is

$$Q_{water} = \dot{m}\, c_p\, \Delta T = \dot{m}\, 4182\, \frac{J}{kg\,°C} \times 5°C = \dot{m}\, 20910$$

Therefore,

$$\dot{m}\, 20910\, \frac{J}{kg} = 2900\, \frac{J}{s}$$

$$\dot{m} = 0.138\, kg/s = 499\, kg/hr$$

For another example of cooling of a plastic in a mold the reader is referred to the last section of this chapter: Special Heat Transfer Problems in Plastics Processing.

HEAT TRANSFER MODES

Temperature differences cause the flow of heat from a high temperature to a low temperature. There are three modes of heat transfer: conduction, convection, and radiation.

The basic microscopic mechanism of conduction is the motion of molecules and electrons. It can occur in solids, liquids and gases. In non-metallic solids the transfer of heat energy is due mainly to lattice vibrations. In metallic solids we have both lattice vibrations and random motions of free electrons. Consequently metals are more conductive than non-metals. In gases, we have mainly random motions of molecules. In liquids we have partly random molecular motions and some sort of vibration of the liquid lattice structure.

Convection is associated with the transport of a mass of liquid or gas. It can be forced i.e. when assisted by a pump or fan, or free (also called natural convection) when the motion of a fluid occurs due to density differences. If there is an electrical heating element at the corner of a room and air is blown onto the element by a fan, this is forced convection. In the absence of a fan the air surrounding the heating element will get hotter, its density will decrease and the air will move upwards causing natural circulation within the room, as the hot air is replenished by colder air, which gets hot and rises again.

Radiation involves electromagnetic waves which are emitted by a body as a result of its temperature. The electromagnetic radiation has a broad spectrum from radio waves to x-rays. Between the two extremes a narrow portion of the radiation spectrum is the visible light and a broader one covers the thermal radiation. The earth is heated by sun's radiation.

CONDUCTION

Consider the solid wall shown in Fig. 1. Temperature T_1 is higher than temperature T_2. Heat flows from the high temperature to the low temperature. If A is the area normal to the direction of heat flow, Fourier's Law states that the amount of heat flow is proportional to the area A, the temperature difference $T_1 - T_2$ and inversely proportional to the thickness of the wall.

$$Q = k \frac{A}{b} (T_1 - T_2)$$

The proportionality constant k is the thermal conductivity measured in W/m °C or Btu/hr ft °F (see Table 1).
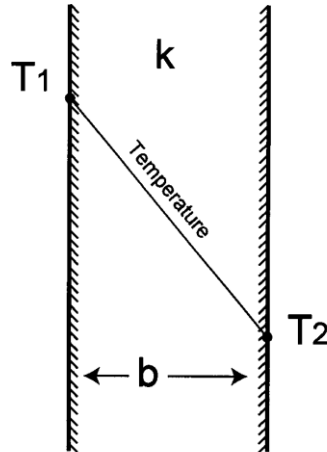
Figure 1
Heat conduction through a solid
wall

A plastic injected into a mold cavity is cooled by heat conduction through the mold wall. In fact, one of the factors considered in choosing mold materials is their thermal conductivity. Aluminum has roughly 5 times the conductivity of steel and Beryllium copper alloys have about 3 times higher conductivity than steel. Higher conductivity means faster heat removal (or addition).

It is interesting to note the similarity between Fourier's law of heat conduction and Ohm's law of electricity. We may write:

$$T_1 - T_2 = \frac{b}{kA} Q \quad \text{Fourier's Law}$$

$$E = R \cdot I \quad \text{Ohm's Law}$$

where E is the voltage (corresponds to $T_1 - T_2$), R the resistance (corresponds to b/kA and I the electric current (corresponds to Q). The quantity b/kA is sometimes referred to as thermal resistance.

Now, let us consider a wall composed of three different materials with thermal conductivities k_1, k_2 and k_3 as shown in Fig. 2. We will have

$$T_1 - T_2 = \frac{b_1}{k_2 A} Q$$

$$T_2 - T_3 = \frac{b_2}{k_2 A} Q$$

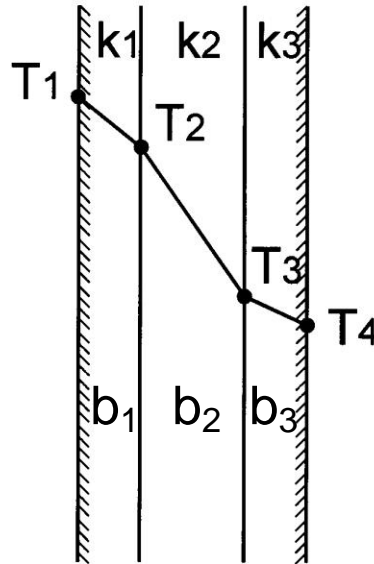$$T_3 - T_4 = \frac{b_3}{k_3 A} Q$$

Figure 2
Heat conduction through a composite wall

Summing up the above expressions, we have

$$T_1 - T_4 = \left(\frac{b_1}{k_1 A} + \frac{b_2}{k_2 A} + \frac{b_3}{k_3 A} \right) Q$$

This equation is similar to the expression of three electrical resistors in series

$$E = (R_1 + R_2 + R_3)\, I$$

We can then write a general expression to calculate the rate of heat flow through a composite wall for more than three layers as follows:

$$Q = \frac{T_1 - T_n}{\dfrac{b_1}{k_1 A} + \dfrac{b_2}{k_2 A} + \dots \dfrac{b_n}{k_n A}}$$

Table 1		
Some typical values of thermal conductivity (k)		
	W/m°C	Btu/hr ft°F
Copper	380	220
Aluminum	204	118
Carbon Steel	43	25
Glass	0.78	0.45
Polymer	0.2	0.115
Water	0.6	0.347
Air	0.025	0.0144

Table 2		
Typical values of convection heat transfer coefficients		
Mode	W/m^2 °C	Btu/hr ft^2 °F
AIR, Free Convection	4 - 28	0.7 - 5
AIR, Forced Convection	4 - 570	0.7 - 100
WATER, Free Convection	284 - 1500	50 - 265
WATER, Forced Convection	284 - 17,000	50 - 3,000
WATER, Boiling	2840 - 57,000	500 - 10,000
STEAM, Condensing	5680 - 113,000	1,000 - 20,000

For the case of composite cylinder (as shown in Fig. 3) the above equation takes the form

$$Q = \frac{T_1 - T_4}{\dfrac{1}{2\pi k_1 L}\, \ell n\left(\dfrac{r_2}{r_1}\right) + \dfrac{1}{2\pi k_2 L}\, \ell n\,\dfrac{r_3}{r_2} + \dfrac{1}{2\pi k_3 L}\left(\dfrac{r_4}{r_3}\right)}$$
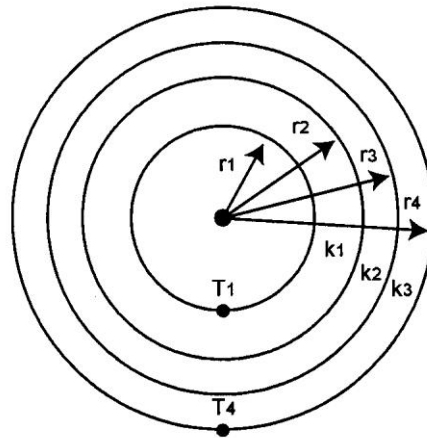

Figure 3
Heat conduction through a composite cylinder.

Unsteady heat conduction involves temperature variations with time. For example, if the surface of a body is suddenly raised to higher temperature how long will it take for the temperature to penetrate inside? Such problems require the solution of differential equations. However, many solutions of such problems for common geometrical shapes appear in the form of charts in specialized texts [4–6].

With the help of Figs. 4 and 5, it is possible to determine how long it will take for the temperature, at the midplane of plate or axis of a cylinder, to reach a certain value, if the surface temperature is suddenly raised or lowered.

On the vertical axes of Figs. 4 and 5, T_m is the (unknown) temperature at the midplane or axis, T_i is the initial temperature of the material (uniform throughout) and T_0 the suddenly imposed surface temperature. On the horizontal axes, the quantity

$$Fo = \frac{k\,t}{\rho\, C_p\, x_m^{\,2}}$$

is the dimensionless Fourier number, and t represents time and x the distance from the surface to the center.
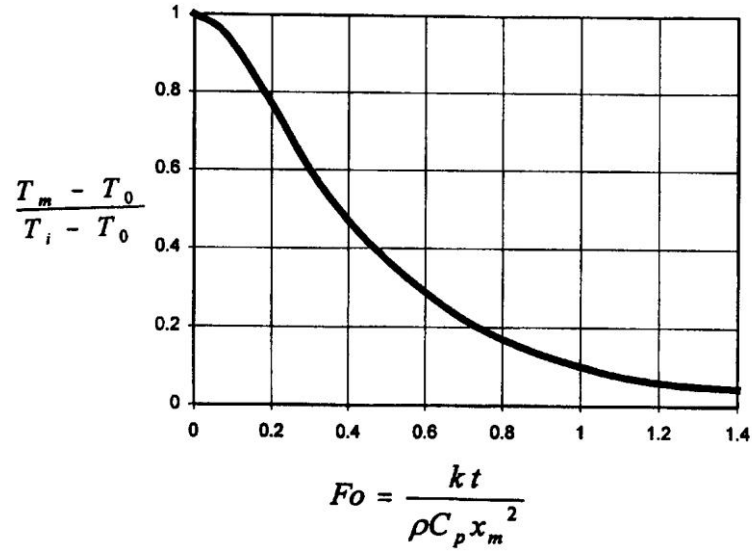
$$Fo = \frac{kt}{\rho C_p x_m^2}$$

Figure 4
Plot for calculating the temperature T_m at the midplane of a plate as
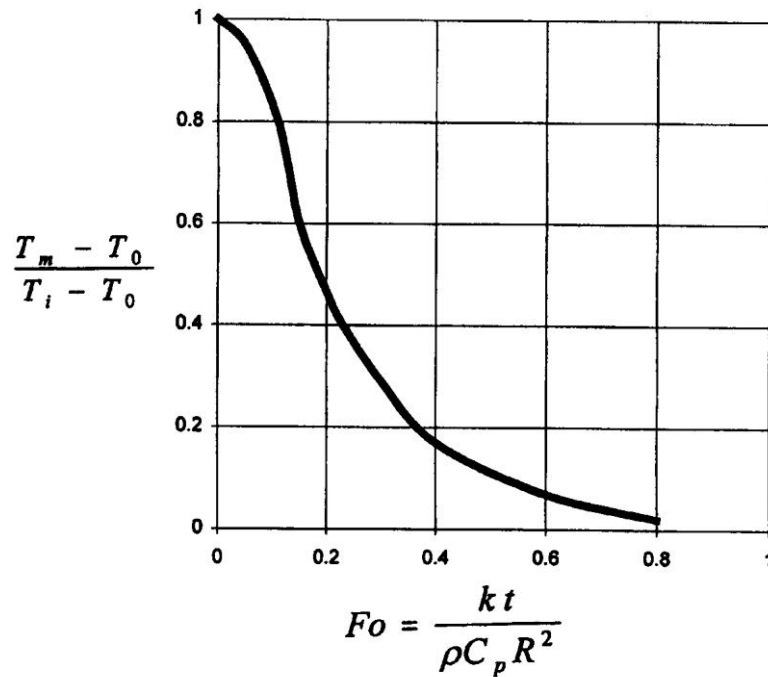a function of time after the two surfaces are suddenly raised to T_0

$$Fo = \frac{kt}{\rho C_p R^2}$$

Figure 5
Plot for calculating the temperature T_m at the axis of a cylinder as
a function of time after the surface tempearature is suddenly raised to T_0

CONVECTION

In most heat transfer problems, we are concerned with solid walls separating liquids or gases from each other. In such cases we usually do not know the temperatures on the wall surfaces, but rather the temperatures of the bulk of fluids on both sides. Careful experiments supported also by theoretical considerations, have shown that the greatest temperature drop is confined within a thin fluid layer attached to a solid surface, as shown in Fig. 6.
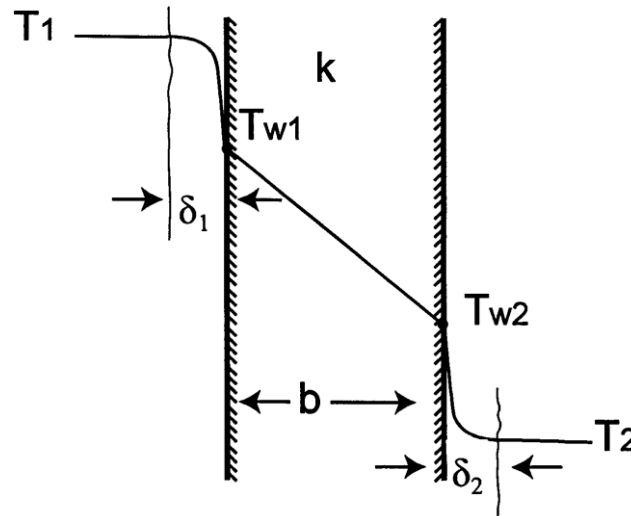

Figure 6
Heat transfer through a wall separating two fluids.

To explain this observation, we may assume that a thin film, of thickness δ, adheres to the wall, whereas outside this film all temperature differences vanish as a result of mixing motions. Within the film heat flow takes place by conduction, as in a solid wall. Thus, in general, we may write

$$Q = \frac{k}{\delta} A(T - T_w)$$

The quantity $k/\delta = h$ is called the <u>heat transfer coefficient</u> and it is an extremely important concept in heat transfer. This simplified model is very useful for practical applications, because the calculation of heat transfer can be made in terms of the heat transfer coefficient:

$$Q = h A (T - T_w)$$

At this point it suffices to say that the heat transfer coefficient depends on the flow conditions and fluid properties. Typical values are given in Table 2. In the next section we will present some correlations that can be used for the more accurate determination of this coefficient.

For the wall separating the two fluids of Fig. 6, we have

$$Q = h_1 A (T_1 - T_{w1})$$

$$Q = \frac{k}{b} A (T_{w1} - T_{w2})$$

$$Q = h_2 A (T_{w2} - T_2)$$

or

$$T_1 - T_{w1} = \frac{1}{h_1 A} Q$$

$$T_{w1} - T_{w2} = \frac{b}{k A} Q$$

$$T_{w2} - T_2 = \frac{1}{h_2 A} Q$$

By summing up

$$T_1 - T_2 = \left(\frac{1}{h_1 A} + \frac{b}{kA} + \frac{1}{h_2 A} \right) Q$$

or

$$T_1 - T_2 = \frac{1}{A} \left(\frac{1}{h_1} + \frac{b}{k} + \frac{1}{h_2} \right) Q$$

The quantity in the brackets is called the <u>total thermal resistance</u>. A more useful concept, however, is the <u>overall heat transfer coefficient</u>, which is defined as follows

$$\frac{1}{U} = \frac{1}{h_1} + \frac{b}{k} + \frac{1}{h_2}$$

For a composite wall separating two fluids (a and b), we have

$$\frac{1}{U} = \frac{1}{h_a} + \frac{b_1}{k_1} + \frac{b_2}{k_2} + \ldots + \frac{1}{h_b}$$

Then, in general, we have:

$$T_1 - T_2 = \frac{1}{A} \frac{1}{U} Q$$

and

$$\boxed{Q = U \ A \ \Delta T}$$

The significance of the overall heat transfer coefficient is that it permits the calculation of the rate of heat flow by multiplying this quantity by the heat exchange area (perpendicular to the heat flow direction) and the temperature difference.

<u>Example</u>

Determine the heat loss through an 8-ft by 4-ft glass window of 4 mm thickness. The inside temperature is assumed to be 24°C (75°F) and the outside temperature is −10°C (14°F). The inside heat transfer coefficient is 5 W/m°C and the outside about 20 W/m°C (due to moderate wind). The thermal conductivity of window glass is 0.78 W/m°C.

<u>Solution</u>

The overall heat transfer coefficient is

$$\frac{1}{U} = \frac{1}{h_1} + \frac{b}{k} + \frac{1}{h_2} = \frac{1}{5 \ W/m^2 \, °C} + \frac{0.004 \ m}{0.78 \ W/m^2 \, °C} + \frac{1}{20 \ W/m^2 \, °C}$$
$$= 0.2 + 0.005 + 0.05 = 0.255 \ m^2 \, °C \, /W$$

$$U = 3.92 \ W/m^2 °C$$

The rate of heat flow is

$$Q = U \ A \ (T - T_2)$$
$$= 3.92 \ W/m^2 °C \times (2.44 \times 1.22 \ m^2) \ (24°C - (-10°C))$$
$$= 3.92 \times 2.98 \times 34 = 397 \ W$$
$$= 1355 \ Btu \, / \, hr$$

Convection Heat Transfer Coefficient Calculation

The most important step in heat convection calculations is the determination of the appropriate heat transfer coefficient. The higher the fluid velocity is, the higher the heat transfer coefficient will be. Numerous correlations have been developed for the calculation of the heat transfer coefficient in terms of dimensionless groups:

Nusselt number is the dimensionless heat transfer coefficient defined as

$$Nu = \frac{h \ D}{k}$$

where the heat transfer coefficient h has dimensions of $W/m^2 {}^\circ C$ or $Btu/hr\ A\ m^2 {}^\circ F$, k the thermal conductivity has dimensions of $W/m {}^\circ C$ or $Btu/hr\ ft {}^\circ F$ and D is a characteristic length (m or ft), such as the diameter of a pipe in which a fluid is flowing.

The Reynolds number (see also the chapter on Fluid Mechanics), is defined as

$$Re = \frac{\rho\ V\ D}{\mu}$$

where ρ is the density (kg/m^3 or lbm/ft^3), V the fluid velocity (m/s or ft/s), μ the viscosity (Pa $\cdot$ s or lb$_f$s/ft^2) and D a characteristic length (m or ft) such as the diameter of a pipe, in which a fluid is flowing.

When the value of the Reynolds number for pipe flow is less than 2100, the flow is streamlined and regular and is called <u>laminar</u>. Above Re = 2100, the flow is highly chaotic and irregular and is said to be <u>turbulent</u>.

The Prandtl number is defined as

$$Pr = \frac{C_p\ \mu}{k}$$

where C_p is the heat capacity (J/kg$^\circ$C or Btu/lbm$^\circ$F), μ the viscosity (Pa $\cdot$ s or lb$_f$s/ft^2) and k the thermal conductivity (W/m$^\circ$C or Btu/hr ft$^\circ$F).

For <u>Laminar Flow</u> in tubes, the following correlation applies:

$$Nu = 3.66 + \frac{0.0668\ (D\,/\,L)\ Re\ Pr}{1 + 0.04\ [(D\,/\,L)\ Re\ Pr]^{2/3}}$$

For <u>Turbulent Flow</u> in tubes, we have

$$Nu = 0.023\ Re^{0.8}\ Pr^n$$

where n = 0.4 for heating and n = 0.3 for cooling.

Numerous other correlations are available in specialized textbooks [4–6]. Generally, for laminar flow Nu depends on the 0.3 power of Re, while for turbulent flow on the 0.8 power of Re.

To calculate the heat transfer coefficient we plug the various quantities in the appropriate correlation to calculate the Nusselt Number Nu. Then, from the definition

$$Nu = \frac{h\ D}{k}$$

we can calculate

$$h = Nu \frac{k}{D}$$

The values given in Table 2 have been calculate from such correlations assuming flow conditions encountered in various practical heat transfer situations (according to reference [4]).

Let us now see how much more effective is turbulent heat transfer over laminar.

Assuming D/L = 1/30, Re = 2000 and Pr = 1 (i.e. water at 200°C), the correlation for laminar flow gives

$$Nu = 3.66 + \frac{0.0668 \left(\dfrac{1}{30}\right) \times 2000 \times 1}{1 + 0.04 \times \left[\dfrac{1}{30} \times 2000 \times 1\right]^{2/3}}$$

$$= 6.37$$

$$\text{and } h = 6.37 \frac{k}{D}$$

We repeat the calculation for turbulent flow assuming Re = 3000 the other quantities remaining the same.

$$Nu = 0.023 \ (3000)^{0.8} \times 1^{0.4} = 13.9$$

$$\text{and } h = 13.9 \frac{k}{D}$$

We see that while the Reynolds number (i.e. velocity) was increased by 50% the heat transfer coefficient was increased by 118% i.e. $(13.9/6.39) \times 100 = 218\% - 100\% = 118\%$. Obviously, to maximize heat transfer rates, we should operate with turbulent flow conditions. For example, by increasing the channel diameter and the coolant velocity, the Reynolds number in the cooling channels of a mold can be increased, beyond the critical value of 2100. The resulting turbulent flow will be much more effective in removing heat from the mold.

Molten polymers are very viscous and the Reynolds number is in the range 0.0001 to 0.01, i.e. polymer melt flows are always laminar. So, for a polymer melt having k = 0.2 W/m°C, $Pr \approx 5 \times 10^6$, and flowing through a 8 mm diameter channel the heat transfer coefficient will be roughly

$$h = 7 \frac{0.2}{0.008} = 175 \ W/m^2 {}^{\circ}C$$

RADIATION

Many types of thermoforming machines use radiation to heat the plastic sheet. Metal rod heaters, halogen tubes and ceramic plates are used frequently. Heater temperatures can reach 700°C and the corresponding radiative heat transfer coefficients can reach 100 W/m^2°C.

Radiation is important wherever very high temperatures are involved. However, radiative effects play a significant role is other less obvious situations. Goose down is known as the most effective insulation for two reasons: reduction of heat conduction by air pockets trapped by clusters of fibers and entrapment of radiation. The goose down fibers have diameter of few microns which is the wavelength of a significant portion of the infrared radiation that escapes from the human body and can go through air pockets and fabrics. Radiation scattering occurs due to equivalence of fiber size and wavelength. Synthetic materials have been manufactured that reportedly match the goose down properties both in reducing conduction and radiation heat loss, again by scattering on micron-sized fibers.

The radiant heat transfer (which is a form of electromagnetic radiation), depends on the absolute temperature and the nature of the radiating surface. This is stated by the Stefan-Boltzmann law for a black body (perfect radiator)

$$Q = \sigma A T^4$$

where T = absolute temperature of the surface of the body in K or R, A the surface area (m^2 of ft^2) and σ the Stefan-Boltzmann constant (σ = 5.669 × 10^{-8} W/m^{2}K^4 or 0.1714 × 10^{-8} Btu/hr ft^2 R^4).

Black bodies are called so, because such materials do not reflect any radiation and appear black to the eye. Thus, a black body absorbs all radiation incident upon it. The "blackness" of a surface to thermal radiation can be quite deceiving insofar as visual observations are concerned [4]. Some visually black surfaces are indeed black to thermal radiation. However, snow and ice appear white and bright, but are essentially "black" in thermal radiation.

When a body with surface temperature T_1 is placed in a closed environment of temperature T_2, the net amount of heat transfer depends on the temperature difference, in the form

$$Q = \sigma A_1 (T_1^4 - T_2^4)$$

For gray bodies, not perfect radiators, we have

$$Q = \varepsilon_1 \sigma A_1 (T_1^4 - T_2^4)$$

where ε_1 = emissivity ($\varepsilon_1 = 1$ for black bodies, $\varepsilon_1 < 1$ for gray bodies). For lampblack $\varepsilon = 0.96$, for oxidized cast iron $\varepsilon = 0.7$, and for polished steel $\varepsilon = 0.1$)

If surface A_1 is not completely enclosed by surface A_2 we must introduce an additional factor to account for the relative geometrical orientation of the two radiating surfaces

$$Q = \sigma \; A_1 \; F_{1-2} \; (T_1^4 - T_2^4)$$

F_{1-2} is called the shape or view factor or angle factor. During the winter in the northern hemisphere, the temperatures are low because the sun's radiation arrives with an unfavorable angle factor.

For practical problems, it is sometimes advisable to define a radiative heat transfer coefficient h_r from the following

$$Q = h_r \; A_1 \; (T_1 - T_2) = \sigma \; A_1 \; F_{1-2} \; (T_1^4 - T_2^4)$$

$$h_r = \frac{\sigma \; F_{1-2} \; (T_1^4 - T_2^4)}{T_1 - T_2} = \sigma \; F_{1-2} \; (T_1^2 + T_2^2) \; (T_1 + T_2)$$

The radiative heat transfer coefficient depends strongly on temperature and is less useful as a concept than the convective heat transfer coefficient. However, it is useful for practical problems involving both convection and radiation, for which we can write the rate of heat flow as

$$Q = (h + h_r) \; A_w \; (T_w - T_\infty)$$

where T_w is the temperature of a wall and T_4 the temperature of a surrounding medium which completely encloses surface A_w.

Due to the fourth power dependence of radiation heat exchange, this mode is more important in very high temperature applications, as in metallurgical operations. In some polymer processing operations, radiation is less important than convection. For example, in blown film extrusion, the film is likely to emerge from the die lips at 200°C (473°K) or so, and a tangentially impinging cooling air jet is likely to have heat transfer coefficient of the order of 100 W/m^2°C. Let us calculate the radiative heat transfer coefficient assuming $F_{1-2} = 1$, and $T_2 = 20$°C (293°K)

$$h_r = \sigma \; F_{1-2} \; (T_1^2 + T_2^2) \; (T_1 + T_2)$$
$$= 5.669 \times 10^{-8} \, W / m^2 \; K^4 \; (473^2 + 293^2) \; (473 + 293)$$
$$= 13.44 \, W / m^2 \cdot C$$

This is significantly lower than the convective heat transfer coefficient (100 W/m^2°C) and may be neglected in some calculations.

HEAT EXCHANGERS

Heat exchangers are devices that transfer heat by convection and conduction between two fluids which are separated by a wall. The automobile radiator is a heat exchanger in which convection

and conduction take place but no radiative heat transfer. Radiative heat exchangers are used in specialized applications in space vehicles and are beyond the scope of most textbooks.

The simplest type of heat exchanger is the double pipe system shown in Fig. 7. It involves tube and annular flow. If the flows are in the same direction, the arrangement is called parallel flow and if the flows are in opposite directions, counterflow. Because of the rather small surface areas which are available for heat transfer, double pipe heat exchangers are used for low to moderate heat transfer rates. For high rates of heat, other types of heat exchangers are used that provide large surface areas. In this category belong the shell-and-tube heat exchangers (that involve several tube passes) and cross-flow configurations (with interconnected passageways).
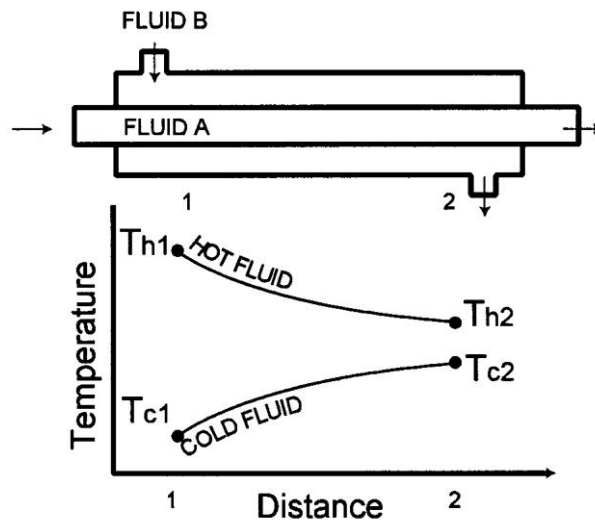

Figure 7
Double-pipe heat exchanger (parallel flow)

For the design and prediction of performance purposes, the concept of the overall heat transfer coefficient is used, which was described earlier in this chapter.

For the double pipe arrangement of Fig. 7, we can write again

$$Q = U \ A \ \Delta T_m$$

where U is the overall heat transfer coefficient, A the heat transfer area, and ΔT_m a suitable average temperature difference between the entering and exiting fluids. It turns out that this average is the so-called log-mean temperature difference (LMTD) which is defined as

$$\Delta T_m = \frac{\Delta T_2 - \Delta T_1}{\ell n \ [\Delta T_2 \ / \ \Delta T_1]}$$

where ℓn represents the natural logarithm having base e = 2.71828, while log has base 10.

The above relationships are valid for both parallel and counterflow heat exchangers. Injection molding machines involving hot oil and cold water are connected counterflow since this arrangement removes roughly 10% more heat from the oil than parallel flow.

For other types of heat exchangers (multipass shell-and-tube; cross flow, etc.), a correction factor is needed

$$\Delta T_m = (LMTD \text{ for counter flow}) \times F_{\text{correction factor}}$$

F is usually between 0.5 and 1.0 and values of this factor for various configurations of heat exchangers can be found in specialized textbooks [4–6].

Heat exchangers may become coated with various deposits present in the flow or corroded. Because of fouling a reduction in heat transfer efficiency is observed. The overall effect is represented by the <u>fouling factors</u>

Definition $$R_f = \frac{1}{U_{\text{fouled}}} - \frac{1}{U_{\text{clean}}}$$

tabulated values of R_f can be found in specialized textbooks [4-6]. For boilers used for long periods of time we may have $R_f = 0.0002$ m^2°C/W, while $U_{\text{clean}} = 3000$ W/m^2°C. Thus

$$0.0002 = \frac{1}{U_{\text{fouled}}} - \frac{1}{3000}$$

$U_{\text{fouled}} = 1875$ W/m^2°C which represents a significant reduction

It is important to keep clean channels and clean circulating water or oil in plastics processing machinery, otherwise fouling will result is significant cooling efficiency reduction.

SPECIAL HEAT TRANSFER PROBLEMS IN PLASTICS PROCESSING

There are two properties of plastics that play a very significant role during their processing: their low thermal conductivity and their high viscosity in the molten state.

The thermal conductivity of most plastics is around 0.2 W/m°C (0.115 Btu/hr A ft°F) which is roughly 200 times smaller than the conductivity of steel and 2000 times smaller than the conductivity of copper. Due to low conductivities, cooling of plastics is slow. In fact, in some processes, such as thick profile extrusion, pipe extrusion and film blowing, cooling might be the output rate limiting step. While the mextruder might be capable of pumping out more product, we might not be able to cool the product fast enough.

The viscosity of molten plastics decreases with the rate of shearing, but it always remains very high, roughly of the order of a million times larger than the viscosity of water. Due to high

viscosity, polymer melts flowing through channels, dies and process equipment tend to raise their temperature by a mechanism of internal friction. Under certain conditions, the frictional heating might result in high temperatures which can cause degradation of the material.

Cooling of a Plastic Plate in a Mold

Let us assume that a HDPE plate of 10 mm thickness initially at 230°C is cooled in a mold and the mold surface is at 30°C. How long will it take for the temperature to reach 90°C at the mid-plane ($x_m = 10/2 = 5$ mm).

We will use Fig. 4 for $T_i = 230$°C, $T_0 = 30$°C, and $T_m = 90$°C. The vertical axis of Fig. 4 is calculated as follows

$$\frac{T_m - T_0}{T_i - T_0} = \frac{90 - 30}{230 - 30} = \frac{60}{200} = 0.3$$

The corresponding Fourier Number is

$$Fo = \frac{k\ t}{\rho\ C_p\ x_m^2} = 0.58$$

Using typical thermal properties for HDPE (see Table 3), we have:

$$\frac{0.25\ t}{780 \times 2300 \times 0.005^2} = 0.58$$
$$t = 104\ \text{seconds}$$

For computer assisted mold cooling calculations and optimization procedures the reader is referred to a thesis [7].

Temperature Rise Due to Frictional Heating

The average temperature rise due to frictional heating (also called viscous dissipation) can be calculated assuming that the process is adiabatic (i.e. there is no heat exchange with the surroundings). This is actually not such a bad assumption for flows through channels and dies.

Assuming that the mechanical work (due to the pressure pushing the molten plastic) is converted into heat, we can easily prove that the temperature rise will be

$$\Delta T = \frac{\Delta P}{\rho\ C_p}$$

So, for a die that has a pressure drop $\Delta P = 4000$ psi (27.58×10^6 Pa) and a molten polymer having $\rho = 780$ kg/m^3 and $C_p = 2300$ J/kg°C, we get

$$\Delta T = \frac{27.58 \times 10^6}{780 \times 2300} = 15.4°C \, (27.7°F)$$

This means that the average temperature of the polymer coming out of the die will be 15.4°C (27.7°F) higher than the temperature coming in.

Power Requirement of a Single-Screw Extruder

This example and the selected thermal properties given in Table 3 are taken from reference [8]. In the extruder, polymer pellets usually coming in at room temperature (i.e. 20°C) are melted and subsequently pumped at the extrusion temperature through the die (i.e. ~200°C, depending on polymer). Most of the energy comes from turning the screw. Of course some energy is supplied by the heating bands around the barrel. In well running extruders usually net energy input occurs in the first section (near the hopper) and net output in the second section (near the die) i.e. the heat generated by the viscous dissipation is really heating the barrel.

The power required by the turning screw is needed to:
- o   Raise the temperature from room temperature to extrusion temperature in the die.
- o   Melt the polymer (heat of fusion).
- o   Pump the molten polymer.

The power required by the turning screw is given by the following expression :

$$Po = \rho \cdot Q \cdot C_p \cdot (T_{out} - T_{in}) + \rho \cdot Q \cdot \Delta h_f + \Delta P \cdot Q$$

Where:

Symbol	Name	Typical Values
$\dot{m}$	Mass Throughput (kg/hr)	
C_p	Heat Capacity, Average (J/kg °C)	1500-3000 J/kg °C
T_{in}	Usually room temperature °C	20°C
T_{out}	Extrusion Temperature in the die °C	200°C -300°C
ρ	Density, average (kg/m$^{3)}$	700-1200 kg/m^3
Q	Volume flow rate (m^3/hr)	
ΔP	Pressure rise (Pa)	10-50 MPa
ΔH_f	Heat of fusion (J/kg)	The heat of fusion can vary from zero (for amorphous polymers) to 300,000 J/kg (for a very crystalline HDPE).

Let us calculate the relative contributions for $\rho Q = \dot{m} = 124.22$ kg / hr of a typical polymer and pressure rise of 30 MPa:

$$Po = 112.4 \times 2500 \times (200 - 20) + 112.4 \times 130{,}000 + 30 \times 10^6 \times \frac{112.4}{760}$$

$$Po = 50{,}580 \text{ kJ/hr} + 14{,}612 \text{ kJ/hr} + 4{,}437 \text{ kJ/hr} = 69{,}629 \text{ kJ/hr}$$

Term	Relative Significance	Contribution (kJ/hr)
Raise temperature to 200°C	Most Important	50,580
Melt the polymer	Somewhat Important	14,612
Pump the molten polymer	Insignificant	4,437
Total Power Requirement		*69,629*

Let us express the final answer in terms of horsepower:

$$Po = 69{,}629 \ \frac{\text{kJ}}{\text{hr}} \times \frac{\text{hr}}{3600 \ \text{s}} = 19.34 \ \text{kW} \Rightarrow Po = 19.34 \ \text{kW} \times \frac{\text{hp}}{0.746 \ \text{kW}} = 25.93 \, \text{hp}$$

So we can size the horsepower of a motor. But we must take into account its efficiency. If we assume 85% efficiency, then

$$MotorPower = \frac{1}{0.85} \ 25.93 \, \text{hp} = 30.50 \, \text{hp} = 40.88 \, \text{kW}$$

SUMMARY

Heat is a form of energy that is transferred from high temperatures to low temperatures by three modes: CONDUCTION, CONVECTION and RADIATION. Conduction through metals e.g. copper, aluminum and beryllium copper alloys is very fast, while through plastics is very slow. The thermal conductivity of aluminum is about 200 W/m°C and of typical plastic about 0.20 W/m°C. This means that heat is conducted one thousand times faster through aluminum than through a plastic. In solid walls the temperature drops linearly from its high value on one side to the low value on the other side. Convection can be natural (free) or forced. Natural convection involves the motion of a gas or liquid due to density differences i.e. hot air is lighter and as it moves upward new air moves near the heating source causing circulation. In forced convection, the movement of a liquid or gas is assisted by a pump or fan. For convective heat transfer calculations it is important to determine the heat transfer coefficient. The heat transfer coefficient increases as the flow velocity increases. Turbulent flow is much more effective than laminar flow for transferring of heat. Radiation is the transfer of heat by electromagnetic waves. It is important whenever high temperatures are involved, such as in plastic sheet heating in thermoforming machines.

In plastics processing, heat is generated within flowing highly viscous melts due to internal friction (viscous dissipation). Most of the energy necessary to heat and melt plastic pellets in an extruder comes from the motor turning the screw. This means we have conversion of mechanical energy into heat.

It is possible to carry out relatively accurate calculations of temperature rise, drop or distribution and the energy requirements, by applying the principle of conservation of energy and the equations of heat conduction, convection and radiation. For conduction the thermal conductivity of the materials involved must be known. For convection the key step is the determination of the heat transfer coefficients, which are usually available as correlations involving the Reynolds number of the flow. Radiation calculations require the material emissivities and the relative geometrical orientation of the radiating surfaces.

REFERENCES

1. Holman, J.P., "Thermodynamics" McGraw-Hill, New York (1988).

2. Sonntag, R.E. and C. Borgnakke, "Introduction to Engineering Thermodynamics", Wiley (2000).

3. Smith, J.M. and H.C. Van Ness, "Introduction to Chemical Engineering Thermodynamics", McGraw-Hill, New York (2001).

4. Holman, J.P., "Heat Transfer", McGraw-Hill, New York (1990).

5. Bejan, A., "Heat Transfer", Wiley, New York (1993).

6. Pitts, D.R. and L.E. Sisson, "Heat Transfer", (Schaum's Outline Series), McGraw-Hill, New York (1998).

7. Reynolds, D.W., "Optimization of Cooling Circuits in Injection Molds", M.Eng. Thesis, Mechanical Engineering, McMaster University, Hamilton, ON, Canada (2000).

8. Vlachopoulos, J. and J.R. Wagner, "The SPE Guide on Extrusion Technology and Troubleshooting", Society of Plastics Engineers, Brookfield, CT (2001).

9. Mark, J.E., "Physical Properties of Polymers Handbook", AIP Press, Woodbury, NY (1996).

Table 3

Selected Thermal Properties.

Polymer	Solid Density* ρ (g/cm^3)	Glass Transition T_g	Melting Point T_m	Usual Melt Processing Range	Melt Density* ρ (kg/m^3)	Thermal Conductivity k (W/m °C) (Btu/h ft °F)	Heat Capacity Cp (J/kg °C) (Btu/lb$_m$ °F)	Heat of Fusion ΔH (J/kg) (Btu/lb)
HDPE	0.941-0.967	-130°C -202°F	130-137°C 266-278°F	160-240°C 320-464°F	780	0.25 0.145	2200-2400 0.52-0.57	210,000-300,000 90-130
LDPE	0.915-0.935	-130°C -202°F	106-112°C 223-234°F	160-240°C 320-464°F	760	0.20 0.115	2200-2400 0.52-0.57	190,000-240,000 80-100
LLDPE	0.910-0.925	-130°C -202°F	125°C 257°F	160-240°C 320-464°F	760	0.20 0.115	2200-2400 0.52-0.57	190,000-240,000 80-100
PP	0.890-0.910	-20°C -4°F	165°C 329°F	180-240°C 356-464°F	730	0.18 0.10	2000-2200 0.48-0.52	210,000-260,000 90-110
PVC (Rigid)	1.30-1.58	80°C 176°F	175°C 347°F	165-205°C 329-401°F	1250	0.17 0.10	1000-1700 0.24-0.41	170,000-190,000 70-80
PS	1.04-1.10	100°C 212°F	amorphous**	180-240°C 356-464°F	1000	0.15 0.09	1300-2000 0.31-0.48	amorphous**
PMMA	1.17-1.20	105°C 221°F	amorphous**	180-230°C 356-446°F	1050	0.19 0.11	1400-2400 0.33-0.57	amorphous**
PET	1.34-1.39	80°C 176°F	265°C 509°F	275-290°C 527-554°F	1160	0.18 0.10	1800-2000 0.43-0.48	120,000-140,000 50-60
ABS	1.01-1.04	105-115°C 221-239°F	amorphous**	200-290°C 392-554°F	990	0.25 0.145	1300-1700 0.31-0.41	amorphous**
Nylon-66	1.13-1.15	90°C 194°F	265°C 509°F	275-290°C 527-554°F	980	0.20 0.115	2400-2600 0.57-0.62	190,000-205,000 80-88
PC	1.2	140°C 284°F	amorphous**	250-305°C 482-581°F	1050	0.22 0.13	1300-2200 0.31-0.52	amorphous**

* Melt densities have been estimated for roughly the mid-temperature of the processing range. See Mark [9] for expressions in the form $\rho = A - BT \pm CT^2$.

** Amorphous resin does not possess crystallinity and consequently no melting point or heat of fusion (i.e. heat to break down crystal structure) can be determined.
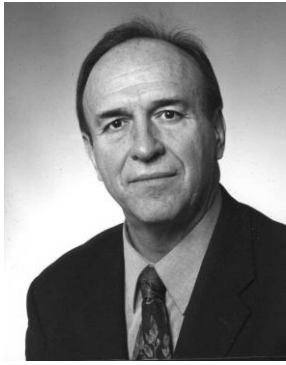
John Vlachopoulos is Professor of Chemical Engineering and Director of the Centre for Advanced Polymer Processing and Design (CAPPA-D) at McMaster University, Hamilton, Ontario, Canada. He earned his Dipl. Ing. from NTU, Athens, Greece and an M.S. and D.Sc. from Washington University, St. Louis, Mo. He is the author of approximately 200 publications on various aspects of plastics processing, frequently lectures at conferences and seminars, and serves as consultant to industry worldwide. He established and is President of Polydynamics, Inc., a company involved in software research, development and marketing. John was elected an SPE Fellow and received the SPE Education Award in 2001.