



UNIVERSITY OF TECHNOLOGY, JAMAICA

FACULTY OF SCIENCE AND SPORT

SCHOOL OF MATHEMATICS AND STATISTICS

Final Examination, Semester 1

Module Name: Engineering Mathematics 3
Module Code: MAT 2034
Date: December 2011
Theory / Practical: Theory
Groups: Eng 2(Electrical / Mechanical)
Duration: Two (2) hours

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Instructions:

1. This question paper consists of four (4) printed pages, which includes a cover page, five (5) questions and a formulae sheet.
2. You are required to **ANSWER ANY FOUR (4)** questions in the answer booklet provided.
3. Full marks will be awarded for full workings / explanations.
4. The use of silent electronic calculators is permitted.
5. Begin the answer to each question on a fresh page and number your solutions carefully.

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

QUESTION 1

a) Differentiate the following, with respect to x :

(i) $y = 4 \tan^{-1}(\sqrt{3x+1})$

(ii) $y = \log_{\sin 3x}(2x^2 + 1)$

(iii) $y = \cos^5(e^{3x})$

[3+4+3 marks]

b) If $z = \frac{8b^5c^2}{\sqrt{d}}$, find the percentage change in z if c is increased by 1.5% and

b and d are decreased by 2% and 3%, respectively.

[5 marks]

QUESTION 2

a) Given the curve $3x^2 \ln(y+1) + 3y - 4e^{\sin x} = 7$, deduce an expression for $\frac{dy}{dx}$.

[6 marks]

b) Given the parametric equations: $y = 3 \cos \theta - \cos^3 \theta$, $x = 3 \sin \theta - \sin^3 \theta$

(i) Show that $\frac{dy}{dx} = -\tan^3 \theta$

(ii) Derive an expression for $\frac{d^2y}{dx^2}$

[5+4 marks]

QUESTION 3

a) Find the stationary points of the function $z = x^3 - 6xy + y^3$.

Classify each stationary point.

[6+4 marks]

b) Find the following integrals:

(i) $\int \left[2x^{-2} + \frac{5}{\sqrt[3]{x^7}} \right] dx$

(ii) $\int \left[7 \sin\left(\frac{3x}{5}\right) + 4e^{7-6x} - \frac{12}{x} \right] dx$

[2+3 marks]

QUESTION 4

- a) Using logarithmic differentiation, find $\frac{dy}{dx}$, given

$$y = \frac{\sqrt[5]{(x^7 - 2x + 1)}}{e^{4x} \sec(3x)}$$

[6 marks]

- b) Given $Z = e^{2x} \cos y - x^2 \ln y$, find expressions for:

(i) $\frac{\partial^2 Z}{\partial x^2}$

(ii) $\frac{\partial^2 Z}{\partial y \partial x}$

(iii) $\frac{\partial^2 Z}{\partial y^2}$

[3+3+3 marks]

QUESTION 5

- a) Find the following integrals:

(i) $\int \left[\frac{3}{\sqrt[3]{(3-10x)^3}} + \sqrt{2x+5} \right] dx$

(ii) $\int \left[5 \sin^7 x \cos x - \frac{4x}{5x^2-6} \right] dx$

(iii) $\int \left[3 \sin\left(\frac{3x}{5}\right) + 5x^2 e^{x^3} - \frac{2}{1+x^2} \right] dx$

[4+4+3 marks]

b) Evaluate $\int_0^{\frac{\pi}{3}} \left[\frac{\sin 3\theta}{4+2\cos 3\theta} \right] d\theta$

[4 marks]

*****END OF EXAM*****

USEFUL FORMULAE

$$\ln(xy) = \ln x + \ln y$$

$$\ln\left(\frac{x}{y}\right) = \ln x - \ln y$$

$$\ln x^n = n \ln x$$

$$\log_a b = \frac{\ln b}{\ln a}$$

$$\frac{d}{dx}(ax^n) = nax^{n-1}$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(uv) = u'v + uv'$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{vu' - uv'}{v^2}$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\frac{d}{dx}(e^{f(x)}) = f'(x)e^{f(x)}$$

$$\frac{d}{dx}(\ln f(x)) = \frac{f'(x)}{f(x)}$$

$$\frac{d}{dx}[(f(x))^n] = n[f(x)]^{n-1} \times f'(x)$$

$$\frac{d}{dx}[f(g(x))] = g'(x)f'(g(x))$$