UNIVERSITY OF TECHNOLOGY, JAMAICA

SCHOOL OF ENGINEERING

FINAL EXAMINATION (SEMESTER 1)

GROUP: ENG 2 **DATE: DECEMBER 2006**

SUBJECT: MATHEMATICS 3 (MAT 2034) **DURATION: 2 HOURS**

INSTRUCTIONS: This paper consists of five questions presented on 3 pages. Answer any four (4) questions. Show all working clearly. [$\Re$ represents the set of real numbers.]

QUESTION # 1

(a) If $Z = x\,Cos(xy)$, show that :

(i) $x\dfrac{\partial Z}{\partial x} - y\dfrac{\partial Z}{\partial y} - Z = 0$

(ii) $x\dfrac{\partial^2 Z}{\partial x\,\partial y} = 2\dfrac{\partial Z}{\partial y} - (x^2 y).Z$ [6 + 3 marks]

(b) The deflection y at the centre of a circular plate suspended at the edge and uniformly loaded is given by $y = \dfrac{kwd^4}{t^3}$, where w = total load, d = diameter of plate, t = thickness and k is a constant. Calculate the approximate percentage change in y if both w and t increase by 2% and 1% respectively, and d decreases by 1.5%.

[Use partial differentiation.] [6 marks]

QUESTION # 2

(a) Find the two stationary points of the function: $Z = 4x^2 + 2y^2 - 8xy + 2x^3$.

For each critical point, determine by the second partial derivative test, whether it corresponds to a relative maximum, relative minimum or saddle point.

[10 marks]

QUES #2 CONTINUED →

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QUESTION # 2 Continued

(b) Given that $R = \dfrac{e^{\left(\frac{4x}{7}\right)} Sin^3(5x)}{\ln x . \sqrt{\left(5x^2 - x\right)^9}}$, find $\dfrac{dR}{dx}$.

[Use logarithmic differentiation or otherwise] [5 marks]

QUESTION # 3

Differentiate the following with respect to x, simplifying your answers as far as possible:

(i) $\log_{1-x}(5x-2)$ [5 marks]

(ii) $Sin\left(e^{(x \ln x)}\right)$ [5 marks]

(iii) $Sec^{-1}\left(4x^8\right)$ [5 marks]

QUESTION # 4

(a) Find the following integrals, simplifying your answers:

(i) $\displaystyle\int\left[\left(x^2 - \frac{1}{3x}\right)^2 - Cos\left(\frac{4-9x}{3}\right) + \frac{7}{2\sqrt[7]{e^{8x}}}\right]dx$ [5 marks]

(ii) $\displaystyle\int\left[5x(6x^2+1)^{10} + \frac{5x}{6x^2+1} - \frac{5}{6\sqrt{x}}\right]dx$ [4 marks]

(b) Find the gradient at the point (0,1) on the curve:

$x^2 \ln(5y) - y^3 e^{2x} + Sin\left(x^2\right) + 1 = 0$ [6 marks]

QUESTION # 5

(a) Evaluate $\dfrac{dy}{dx}$ *and* $\dfrac{d^2y}{dx^2}$ at the point where $\theta = \pi$

$x = Cos\theta + Sin\theta, \quad y = Sin\theta - Cos\theta$ [7 marks]

QUES. # 5 CONTINUED →

QUESTION # 5 Continued

(a) Evaluate the following integrals:

(i) $$\int_0^{\pi}\left[\frac{Cos\left(\frac{x}{2}\right)}{1\ +\ 4Sin\left(\frac{x}{2}\right)}\right]dx$$ [4 marks]

(ii) $$\int_1^{3}\left(\frac{6x^4-3x^2+2x}{3x^3}\right)dx$$ [4 marks]

************END OF EXAM************

USEFUL FORMULAE

$$Sin^2A+Cos^2A=1$$

$$Tan^2A+1=Sec^2A$$

$$Cot^2A+1=Cosec^2A$$

$$\frac{d}{dx}Sec(x)=Sec(x)Tan(x)$$

$$\frac{d}{dx}Cosec(x)=-Cosec(x)Cot(x)$$